ABSTRACTS OF ARTICLES DEPOSITED AT VINITI*

TIME – TEMPERATURE DEPENDENCE DETERMINATION OF THERMOPHYSICAL PROPERTIES OF PLASTICS UNDER CONDITIONS OF THERMAL DESTRUCTION

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UDC 678:536.6

It is proposed that the thermophysical properties of composition plastics be determined under the conditions of thermal decomposition related by the degree of the decomposition process completion which in turn can be associated with the density of the material. On the basis of processing a large number of thermogravimetric experiments it was established that the density of these materials changes over a comparatively small region; the extent of the change depends on the ratio of the components. The upper boundary of this region, conditionally named the boundary of the instantaneous density values, can be determined from the experimental results with a heating rate of 1-10°/sec using low thickness specimens of the material in which the condition of uniform (within a given accuracy) temperature distribution has been assured. The lower boundary is determined using the results of experiments in which the specimens are subjected to long heating upon reaching the equilibrium state, and the complete accomplishment of the destruction process at each of the intermediate temperatures occurs. Analogous boundaries occur for the region of changes in the thermal-conductivity coefficient. It is proposed that the connection between the density and the thermal-conductivity coefficient be established using the simplest approximating functions or on the basis of the Maxwell equation.

The determination of the indicated boundaries substantially simplifies the experimental determination and the description of the thermophysical properties of plastics under the conditions of thermal destruction. Examples of experimental data processing by the proposed methods and the kinetic characteristics of some vitreous plastics determined at heating rates of over 100°/sec and under isothermal conditions are presented.

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EFFECT OF VARIOUS FACTORS ON DESTRUCTION OF POLYACRYLAMIDE SOLUTION

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UDC 532.517.4:532.135

The article reports the results of experiments regarding the explanation of reasons which bring about the destruction of polymers. For this end, several series of experiments were conducted on determining the effectiveness of polyacrylamide solutions of various concentrations in lowering the hydraulic resistances depending on the duration of their movement at constant and variable speeds, and other factors.

The experiments were conducted in a laboratory apparatus, operating in a closed loop. The range of Reynolds number changes upon the movement of the solutions was from $2 \cdot 10^4$ to $1.3 \cdot 10^5$.

Before conducting the basic series of experiments on the study of the anomalous properties of polyacrylamide solutions, experiments were carried out to establish the duration of the apparatus operation at which all of the parameters under study would be measured under identical conditions.

The effect of concentration on the changes of the hydraulic resistances was studied in the first series of experiments. The second series established the mechanism of the hydraulic resistance changes due not only to the solution velocity but also to time.

The third series was a repetition of the first with one difference: The concentration was increased by adding fresh polyacrylamide portions to the solution which had already been subjected to the test.

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The experiments showed that all of the examined factors contribute to a certain degree to polymer destruction, which impairs the ability of the solution to lower the resistance of the turbulent friction.

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LOW-PRESSURE FLOW OF WET STEAM IN A LAVAL NOZZLE

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UDC 533.6.011

Measurements are reported on a boundary layer in a nozzle, particularly in over-expansion mode; a Laval nozzle of axially symmetrical conical form is used with fairly large f. The conicities of the expanding parts were 1:3.

Figure 1 shows the distribution of the total pressure as measured by a total-pressure probe at various points for five such nozzles. Condensation steps occur near the throat in the expanding sections; all the measurements were performed far from the throat, i.e., behind the condensation step. These nozzles were enclosed in vacuum. No special cooling or heating was employed.

The thickness of the dynamic boundary layer in each section may be judged from the distribution of P_0' along the section. The isentropic core has constant P_0', whereas P_0' varies in the boundary layer. It is clear that the boundary layers make up much of the flow in the nozzles.

The following approximate relationship was obtained for M = 4.2-5:

$$\frac{\delta}{d_c} = 10.2 - 1.9 \lg \mathrm{Re}.$$

The thickness δ of the boundary layer was the value of the transverse coordinate for each section of the nozzle reckoned from the wall such that P_0' differed from that in the isentropic core by 2-3%. The values of Re and M

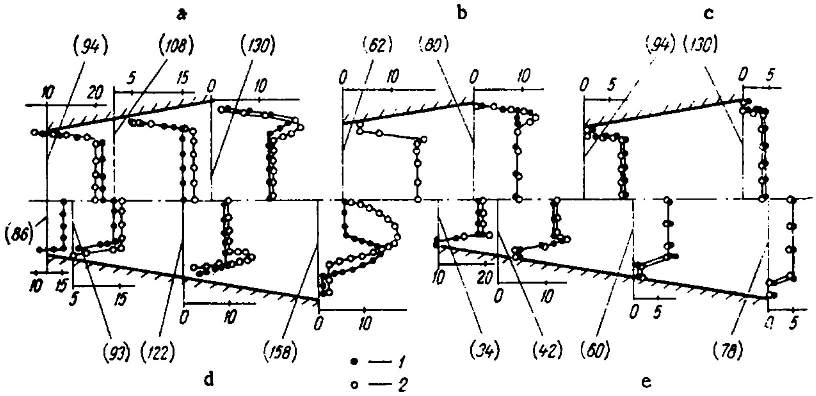

Fig. 1. Distribution of total pressures $10^3 \cdot P_0'/P_0$ in cross sections of nozzles: a) 1) d_c = 9.6 mm, d_e = 130 mm, $P_b = 5.54 \cdot 10^5$ Pa, T_0 = 464°K, P_m = 693 Pa; 2) 9.6 and 130, $3.76 \cdot 10^5$, 447, 466; b) 1) 5.1 and 80, $5.73 \cdot 10^5$, 469, 266; 2) 5.1 and 80, $3.84 \cdot 10^5$, 458, 266; c) 1) 5.1 and 130, $5.6 \cdot 10^5$, 458, 129; 2) 5.1 and 130, $3.75 \cdot 10^5$, 441, 100; d) 1) 7.4 and 158, $5.86 \cdot 10^5$, 462, 573, 2) 7.4 and 158, $3.8 \cdot 10^5$, 445, 532; e) 1-3 and 78, $5.67 \cdot 10^5$; 443, 160; 2-3 and 78, $3.76 \cdot 10^5$, 429, 113. Sections of nozzles denoted by the diameter (mm) of the nozzle in parentheses. The abscissa shows $10^3 \cdot P_0'/P_0$, while the ordinates are lines denoting the corresponding sections (the length of a line from the axis to the wall corresponds to half the value given in parentheses).

were determined from the parameters of a one-dimensional flow calculated on the basis of isentropic flow of moist steam in thermodynamic equilibrium; the characteristic dimension occurring in Re was taken as the diameter of the nozzle section. The dynamic viscosity was taken for the vapor phase.

The distribution of the static pressure along the axis of the nozzle in over-expansion mode shows that the flow is decelerated in diffuse shock waves; the static pressure increases in a certain zone of considerable width. The reduction in the pressure level causes the extent of the increased-pressure zone to increase, since the viscosity then has more effect. The flow is not subsonic beyond the increased-pressure zone, since there is a fall in pressure and the flow again accelerates. The first increased-pressure zone is followed by a second one if n is large.

NOTATION

f	$= d_e/d_c$;
d_e	is the diameter of exist section of nozzle;
d_c	is the diameter of critical section;
M	is the Mach number;
P_0'	is the total pressure measured by a total-pressure probe;
P_0 and T_0	are the pressure and temperature of the steam at the inlet to the nozzle;
P_m	is the pressure in the medium flowing from the nozzle;
δ	is the thickness of the dynamic boundary layer;
Re	is the Reynolds number;
$n = P_m/P_e$	is the degree of over-expansion;
P_e	is the static pressure at entry to the nozzle, as determined from the condition for one-dimensional isentropic flow of thermodynamically equilibrium moist steam.

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A MASS-TRANSFER PROBLEM IN MATERIALS WITH "MEMORY"

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UDC 66.021.3

In complicated cases of a complex mass transfer accompanied by the interaction of the transfer stream with the structure of the substance, the effects of the concentration field evolution character change are observed during the periodic change of the initial and the boundary conditions with time. The linear mass-transfer equations cannot describe such effects because the latter carry the "memory" character. The cyclic adsorption–desorption processes, polymerization processes, etc., often display effects which may be described in terms of mass transfer with a "memory."

For mass-transfer processes, the relationship between the stream and the concentration gradient of the form

$$\bar{j} = -\int_0^\infty D(\theta) \left| \frac{\partial c(r, \tau-\theta)}{\partial r} \right|^{n-1} \frac{\partial c(r, \tau-\theta)}{\partial r} \, d\theta, \; n \leqslant 1.$$

is examined.

This relationship leads to the mass-transfer equation of the form

$$\beta(0) \frac{\partial c}{\partial \tau} + \int_0^\infty \beta'(\theta) \frac{\partial c(r, \tau-\theta)}{\partial \tau} \, d\theta = \frac{D(0)\, n}{r^2} \left| \frac{\partial c}{\partial r} \right|^{n-1} \frac{\partial}{\partial r} \left[r^2 \frac{\partial c}{\partial r} \right] +$$

$$+ \int_0^\infty \frac{D'(\theta)\, n}{r^2} \left| \frac{\partial c(r, \tau-\theta)}{\partial r} \right|^{n-1} \frac{\partial}{\partial r} \left[r^2 \frac{\partial c(r, \tau-0)}{\partial r} \right] d0, \; n \leqslant 1 \tag{1}$$

with boundary conditions

$$c(r)|_{\tau=0} = 0;\ c(\tau)|_{r=1} = 1;\ \left.\frac{\partial c}{\partial r}\right|_{r=0} = 0. \tag{2}$$

The formulated problem (1), (2) was solved for n = 1 using the operational method. For a more complicated case when n < 1, a fragmentary linear approximation is used, which consists of the fact that the region of the gradient change is broken up into partial segments, and the solution of the nonlinear equation (1) is replaced by the solution of linear equations in each of the segments.

NOTATION

c	is the concentration;
j	is the stream of substance;
r	is the coordinate;
τ	is the time;
θ	is the time lag;
$D(\theta)$	is the relaxation function of the substance stream;
$\beta(\theta)$	is the energy-concentration–relaxation function.

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DETERMINATION OF ELECTRON CONCENTRATION IN HIGHLY DISPERSED AEROSOLS WITH ELECTRON-EMITTING PARTICLES

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UDC 537.562

The concentration of electrons in a highly dispersed aerosol ($\lambda \gg R$) the particles of which emit photo- or thermoelectrons has been determined under the condition that the thermal movement energy of the atoms, as well as the absolute value of the potential energy of the electron in the electric field of a positively charged aerosol particle is much lower than the average kinetic energy of the electrons (λ is the average length of the free path of an electron upon its dispersion on the atoms; R is the radius of the aerosol particle). In a quasi-stationary approximation, the kinetic equation for the case $\lambda\pi R^2 N_p \ll 1$ has the form

$$\frac{m}{M}\,\frac{1}{V^2}\,\frac{d}{dV}\left(\frac{V^4}{\lambda}\,f\right) + V\pi R^2 N_p (f_0 - f) = 0,$$

and its solution when λ = const is the function

$$f(V) = (2\gamma/V^4)\int_V^\infty X^3 f_0(X)\exp\left[-\int_V^X (2\gamma dy)/y\right] dX,$$

where $\gamma = (M\lambda\pi R^2 N_p/2m)$. Here N_p is the concentration of the aerosol particles; V and m are the velocity and the mass of the electron, respectively; f and f_0 are the distribution function of the absorbed and the emitted electrons, respectively; and M is the mass of the atom.

In the above kinetic equation, in the integral of collisions of electrons with the atoms presented in the differential form, the member containing the gas temperature is omitted; the electron–electron collisions are completely disregarded; and the aerosol particles are examined as neutral molecules that absorb and emit electrons. If the particles emit only thermoelectrons, then

$$f_0 = N_0 (m/2\pi kT_p)^{3/2} \exp[-mV^2/2kT_p],$$
$$N_0 = 2(2\pi mkT_p/h^2)^{3/2} \exp[-W/kT_p],$$

where N_0 is the electron concentration in an isothermal aerosol when the gas temperature is equal to the temperature of the particles, T_p; h and k are the Planck and Boltzmann constants, respectively; W is the work expended upon discharging the aerosol particle material.

Substituting the above expression for $f_0(V)$ into the formula for $f(V)$ and carrying out the appropriate calculations, expressions for concentration N_e and for the average kinetic energy of the aerosol electrons $\bar{E}$ can be obtained:

$$N_e = N_0\gamma/(\gamma - 0.5), \quad \bar{E} = 1.5\, kT_p\,(\gamma - 0.5)/(\gamma + 0.5).$$

As an example, let us examine an actual aerosol with parameters $N_p = 10^{13}\ m^{-3}$, $R = 10^{-6}$ m, $T_p = 1850°K$, $2m/M = 5.5 \cdot 10^{-4}$, $\lambda = 2.41 \cdot 10^{-5}$ m, for aluminum oxide particles $W = 7.5 \cdot 10^{-19}$ J. Then $\gamma = 1.375$, $N_0 = 6.08 \cdot 10^{13}\ m^{-3}$, $N_e = 9\,55 \cdot 10^{13}\ m^{-3}$.

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EFFECT OF CHANNEL SURFACE CONFIGURATION ON FLOW CHARACTERISTICS AT LOW REYNOLDS NUMBERS

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UDC 532.526.3

The solution to the problem of the viscous fluid moving in a flat undulating channel was examined. The equation for the surface of the walls is given in the form of periodic functions. Assuming that the relative amplitude of a wave is less than unity, we seek the solution of the movement equation in the flow functions by the method of expansion into Taylor series, limiting ourselves to the examination of linear approximations. In the zero approximation, the solution coinciding with the known solution was obtained. In the first approximation, taking into account only the members containing the relative amplitude to the first power, the problem is reduced to the solution of the system of equations

$$\begin{gathered} \partial^4\psi/\partial x^4 + 2\partial^4\psi/\partial x^2\partial y^2 + \partial^4\psi/\partial y^4 = 0, \\ \psi = 0;\ \partial\psi/\partial y + 3/2\, Q \sin(kx + \varphi) = 0 \ \text{at} \quad y = -1, \\ \psi = 0;\ \partial\psi/\partial y = 0 \ \text{at} \quad y = 1. \end{gathered} \tag{1}$$

The exact solution of the system of equations (1) was determined, and the simplified calculation dependences were presented, in particular for the flow function, in the form

$$\psi_{(1)} = 3/2Q\,(1 + y)\exp\left[-k(1 + y)\right]\sin(kx + \varphi). \tag{2}$$

The quantitative and the qualitative analyses of the results allow to make the conclusion about the fact that the disturbances due to undulation are localized at the corresponding wall, with the thickness of the disturbed area on the order of one wavelength, while the amplitude affects merely the intensity of the disturbance.

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Original article submitted March 28, 1977.

RADIANT HEAT TRANSFER IN A SYSTEM OF CONCENTRIC SPHERES

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UDC 536.3

In the article the problem of the radiant heat transfer is solved by the zonal method. The nonisothermal gas volume is divided in this case into p isothermal volume zones (spherical rings). For the determination of the direct mutual transfer surfaces between zones the following dependences are proposed:

a) between volume zones ($i > j$; $i = 2, \ldots, p$; $j = 1, 2, \ldots, p-1$):

$$\overline{g_i g_j} = F_i (A_{i,j} - A_{i,j+1}) - F_{i+1} (A_{i+1,j} - A_{i+1,j+1}) - (F_i - F_{\text{м}}) \times \times (A^{\text{м}}_{i,j} - A^{\text{м}}_{i,j+1}) + (F_{i+1} - F_{\text{м}}) (A^{\text{м}}_{i+1,j} - A^{\text{м}}_{i+1,j+1}), \tag{1}$$

where j and $j+1$, i and $i+1$ are the spherical surfaces limiting the volumes g_j and g_i, respectively, whereby the area of the sphere $F_i < F_j$, $F_i > F_{i+1}$, $F_j > F_{j+1}$; k and m are the internal and the external spheres;

b) between the surface zone j and the volume zone i:

$$\overline{g_i s_j} = F_{i+1} A_{i+1,j} - F_i A_{i,j} + (F_i - F_{\text{м}}) A^{\text{м}}_{i,j} - (F_{i+1} - F_{\text{м}}) A^{\text{м}}_{i+1,j} \quad (i \geqslant j;\ F_i \leqslant F_j), \tag{2}$$

$$\overline{g_i s_j} = F_j (A_{j,i} - A_{j,i+1}) \quad (j \geqslant i+1;\ F_j \leqslant F_{i+1}); \tag{3}$$

c) volume g_i with itself:

$$\overline{g_i g_i} = 4 V_i K_a - \overline{g_i s_k} - \overline{g_i s_{\text{м}}} - \sum_j \overline{g_i g_j} \quad (j \neq i), \tag{4}$$

where V_i is the volume of the spherical ring g_i.

In $\overline{g_i g_j}$, $\overline{g_i g_i}$, $\overline{g_i s_j}$ components can be isolated which take into account that part of radiation of the volume zone g_i which passes through the internal volume-limiting surface, is weakened by passing through the other volume zones and is only then absorbed by the volume zone g_j, the radiating volume zone g_i itself, and the exterior (enveloping the spherical ring g_i) surface zone s_j, respectively:

$$\overline{g_i g_j''} = (F_{i+1} - F_{\text{м}}) (A^{\text{м}}_{i+1,j} - A^{\text{м}}_{i+1,j+1}) - (F_i - F_{\text{м}}) (A^{\text{м}}_{i,j} - A^{\text{м}}_{i,j+1}) + (F_i - F_{i+1}) (A^{i+1}_{i,j} - A^{i+1}_{i,j+1}), \tag{5}$$

$$\overline{g_i g_i''} = (F_{i+1} - F_{\text{м}}) (2 A^{\text{м}}_{i+1,i} - A^{\text{м}}_{i+1,i+1}) - (F_i - F_{\text{м}}) A^{\text{м}}_{i,i} + (F_i - F_{i+1}) A^{i+1}_{i,i}, \tag{6}$$

$$\overline{g_i s_j''} = (F_i - F_{\text{м}}) A^{\text{м}}_{i,j} - (F_{i+1} - F_{\text{м}}) A^{\text{м}}_{i+1,j} - (F_i - F_{i+1}) A^{i+1}_{i,j}; \tag{7}$$

d) between surface zones

$$\overline{s_i s_j^{\text{м}}} = (F_i - F_{\text{м}}) (1 - A^{\text{м}}_{i,j}) \quad (i \geqslant j;\ R_i \leqslant R_j;\ i = 1, 2, \ldots, p;\ j = 1, 2, \ldots, p), \tag{8}$$

$$\overline{s_i s_j} = F_i (1 - A_{i,j}) \quad (i > j;\ R_i < R_j;\ i = 2, \ldots, p+1;\ j = 1, 2, \ldots, p). \tag{9}$$

The absorption capabilities of the medium at a constant absorption coefficient K_a for black radiation are determined from the following dependences:

a) between the internal surfaces of the spheres in the presence of an internal concentric sphere not conducting radiation ($i \geq j$; $R_i \leq R_j$; $i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, p$):

$$A^{\text{м}}_{i,j} = 1 - \frac{R_i^2}{R_i^2 - R_{\text{м}}^2} \int\limits_{(R_{\text{м}}/R_i)^2}^{1} \exp\left[-K_a R_j \left(\sqrt{1 - (R_i/R_j)^2 t} + \frac{R_i}{R_j} \sqrt{1 - t}\right)\right] dt = 1 - \frac{R_j^2}{R_i^2 - R_{\text{м}}^2} \int\limits_{(R_{\text{м}}/R_j)^2}^{(R_i/R_j)^2} \exp\left[-K_a R_j \left(\sqrt{(R_i/R_j)^2 - t} + \sqrt{1 - t}\right)\right] dt; \tag{10}$$

b) between the internal surface of the external sphere and the external surface of the internal sphere i ($i \geq j$; $i = 1, 2, \ldots, p+1$; $j = 1, 2, \ldots, p+1$; $R_i \leq R_j$):

$$A_{i,j} = 1 - \frac{R_j^2}{R_i^2} \int\limits_0^{(R_i/R_j)^2} \exp\left[-K_a R_j \left(\sqrt{1 - t} - \sqrt{(R_i/R_j)^2 - t}\right)\right] dt. \tag{11}$$

By appropriate substitution of the indices, all sought for absorption capabilities can be determined from Eqs. (10)-(11).

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OPTIMAL USE OF THE PSYCHOMETRIC TEMPERATURE DIFFERENCE IN REFRIGERATION

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UDC 621.568

The best method of using air cooling to produce cold has been established by discussing the processes occurring when unsaturated air flows around a plate wetted on one side. Analysis has been made of the temperature distributions on the dry and wet sides. The limiting temperature drop on the dry side has been calculated and measured, and this is equal to the dew point. However, the air tends to reach the wet side while still having some capacity to take up water vapor, and this can be utilized fully if the air flow is properly controlled. Maintenance of the heat balance requires recycling of some part of the cooled air, while the ratio of the main flow rate to the total flow rate may be represented as the specific flow rate of the main flow M.

An apparatus that cools unsaturated air to the dew point with full use of the psychometric temperature difference is called ideal; thermodynamic analysis of the ideal model shows that the maximum effect can be obtained by the evaporation of water at very low cost and by means of reversible processes.

The specific flow rate in the main flow is determined in an ideal model only by the parameters of the outside air, and this indicates the scope for using air for cooling by means of water evaporation. A $M^{id}-t$ plot enables one to determine the optimum balance between the two flows in relation to the climatic conditions.

A method has also been given for evaluating the thermodynamic performance of an indirect-evaporation system, particularly as regards the energy of the cold produced by an actual equipment in relation to that of the ideal model.

The degree of thermodynamic perfection in a real apparatus is the product of the degree of use of the air and the efficiency of the apparatus $\eta = \lambda E_p^2$.

A relationship has been derived between the performance factor and the energy change in the flow, $E_p = \sqrt{(e_2 - e_1)/(e_p - e_1)}$.

The ideal model has also been used in the RKV indirect-regenerative cooler; tests on this have shown that it is possible to cool the air below the initial wet-bulb temperature, and the degree of thermodynamic perfection of the apparatus is 2.5 times that of existing indirect-evaporation coolers.

NOTATION

t	is the temperature, °C;
G	is the air flow rate, kg/sec;
$M = G_0/G_f$	is the specific flow rate in main air flow;
η	is the degree of thermodynamic perfection;
E	is the energy of heat flux, kW;
$\lambda = M^a/M^{id}$	is the degree of use of air;
E_p	is the performance factor;
e	is the specific energy of substance J/kg.

Indices

1	are the initial parameters;
2	are the parameters at outlet from dry cavity;
d	is the dew point;
id	is the ideal model;
a	is the actual cooler;
w	is the water.

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SOLUTION OF CERTAIN THREE-DIMENSIONAL HEAT-CONDUCTION PROBLEMS IN BODIES OF COMPLEX SHAPE

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UDC 536.24

We present a method for reducing a three-dimensional steady-state heat-conduction problem in cylindrical bodies with longitudinal cooling channels to a series of two-dimensional problems. It is assumed that the internal heat source as well as the specified boundary conditions of the second kind can be represented by finite expansions in the longitudinal coordinate, or are well approximated by expressions of the form $F(x, y, z) = \sum_i F_i(x, y)\xi_i(z)$. We seek the temperatures of the body and the gas coolant in the form of similar expansions one degree higher than the expansion of the internal heat source. The thermal conductivity is assumed constant.

Under these assumptions a three-dimensional problem is reduced to n + 2 two-dimensional problems (where n is the order of the expansion of the internal heat source) of the form

$$-\Delta_{x,y}t_k(x, y, z_0) = \varphi_k(x, y, z_0) + (k+2)(k+1)\,t_{k+2}(x, y, z_0),$$

$$\frac{\partial t_k}{\partial \nu} = a_{ik}(x, y, z_0)\Big|_{S_i}, \quad i = 1, 2, \ldots, N,$$

$$\frac{\partial t_k}{\partial \nu} = \delta_i(c_{ik} - t_k)\Big|_{S_i}, \quad i = N+1, \ldots, M,$$

$$c_{i,k+1} = \alpha_i \frac{\int\limits_{S_i}(t_k - c_{ik})\,ds}{(k+1)\,c_pQ_i}, \quad i = N+1, \ldots, M,$$

$$k = 0, 1, \ldots, n+1.$$

Here $\Delta_{x,y}$ is the Laplacian operator, ν is the outward normal, Q_i is the flow rate of the coolant in the i-th channel, α_i is the heat-transfer coefficient, c_p is the specific heat of the coolant, and the c_{ik} are expansion coefficients.

These problems can be solved in sequence from k = n + 1 to k = 0, since they are coupled through the boundary conditions.

It is proposed to solve the two-dimensional problems for circular cooling channels by the variational method using a set of appropriate coordinate functions, including the logarithms of the local channel radii, which permits a good approximation of the temperature distribution in the region considered. A numerical solution of a model two-dimensional problem obtained by the proposed method is compared with solutions of the same problem obtained by other methods.

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TEMPERATURE DISTRIBUTION IN HOLLOW BODIES FOR GENERAL BOUNDARY CONDITIONS

A. G. Gorelik

UDC 536.21

We consider the problem of calculating the temperature distribution in a plate, a hollow cylinder, and a hollow sphere with internal heat sources for general boundary conditions of the third kind with variable heat fluxes across the boundaries and varying coolant temperatures:

$$\frac{\partial t}{\partial \mathrm{Fo}} = \frac{\partial^2 t}{\partial r^2} + \frac{k-1}{r} \cdot \frac{\partial t}{\partial r} + \frac{R^2}{\lambda} Q(r, \mathrm{Fo}), \quad \frac{r_0}{R} \leqslant r \leqslant 1, \ \mathrm{Fo} > 0, \tag{1}$$

$$-\frac{\partial t\left(\frac{r_0}{R}, \mathrm{Fo}\right)}{\partial r} = \frac{q_0(\mathrm{Fo}) R}{\lambda} + \mathrm{Bi}_0 \left[\psi_0(\mathrm{Fo}) - t\left(\frac{r_0}{R}, \mathrm{Fo}\right)\right], \tag{2}$$

$$\frac{\partial t(1, \mathrm{Fo})}{\partial r} = \frac{q_1(\mathrm{Fo}) R}{\lambda} + \mathrm{Bi}_1 [\psi_1(\mathrm{Fo}) - t(1, \mathrm{Fo})], \tag{3}$$

$$t(r, 0) = \varphi(r) \tag{4}$$

(k = 1, 2, 3 for a plate, cylinder, and sphere, respectively).

The problem is solved by using integral transforms with the following kernels:

$$V_0(\mu r) = \mathrm{Bi}_0 \sin\left[\mu\left(r - \frac{r_0}{R}\right)\right] + \mu \cos\left[\mu\left(r - \frac{r_0}{R}\right)\right] \qquad \text{plate} \tag{5}$$

$$V_0(\mu r) = \left[\mathrm{Bi}_0 Y_0\left(\mu \frac{r_0}{R}\right) + \mu Y_1\left(\mu \frac{r_0}{R}\right)\right] J_0(\mu r) - \left[\mathrm{Bi}_0 J_0\left(\mu \frac{r_0}{R}\right) + \mu J_1\left(\mu \frac{r_0}{R}\right)\right] Y_0(\mu r) \qquad \text{cylinder} \tag{6}$$

$$V_0(\mu r) = \left\{\mu \cos\left[\mu\left(r - \frac{r_0}{R}\right)\right] + \left(\mathrm{Bi}_0 + \frac{R}{r_0}\right) \sin\left[\mu\left(r - \frac{r_0}{R}\right)\right]\right\} \Big/ \mu \qquad \text{sphere} \tag{7}$$

Here μ stands for the roots of the characteristic equations

$$(\mathrm{Bi}_0 \mathrm{Bi}_1 - \mu^2) \sin\left[\mu\left(1 - \frac{r_0}{R}\right)\right] + (\mathrm{Bi}_0 + \mathrm{Bi}_1) \cos\left[\mu\left(1 - \frac{r_0}{R}\right)\right] = 0 \qquad \text{plate} \tag{8}$$

$$\left[\mathrm{Bi}_0 J_0\left(\mu \frac{r_0}{R}\right) + \mu J_1\left(\mu \frac{r_0}{R}\right)\right] [\mathrm{Bi}_1 Y_0(\mu) - \mu Y_1(\mu)] -$$
$$-\left[\mathrm{Bi}_0 Y_0\left(\mu \frac{r_0}{R}\right) + \mu Y_1\left(\mu \frac{r_0}{R}\right)\right] [\mathrm{Bi}_1 J_0(\mu) - \mu J_1(\mu)] = 0 \qquad \text{cylinder} \tag{9}$$

$$\left[(\mathrm{Bi}_1 - 1)\left(\mathrm{Bi}_0 + \frac{R}{r_0}\right) - \mu^2\right] \sin\left[\mu\left(1 - \frac{r_0}{R}\right)\right] + \mu\left(\mathrm{Bi}_1 + \mathrm{Bi}_0 + \frac{R}{r_0} - 1\right) \cos\left[\mu\left(1 - \frac{r_0}{R}\right)\right] = 0 \qquad \text{sphere} \tag{10}$$

After taking the inverse transform the solution for a hollow sphere can be written in the form

$$t(r, \mathrm{Fo}) = 4 \sum_{n=1}^{\infty} \frac{\mu_n^2 V_0(\mu_n r)}{\Phi(\mu_n)} \left[\frac{1}{\mu_n} \int_{r_0/R}^{1} r\varphi(r) V_0(\mu_n r)\, dr + \frac{R}{\lambda} \times\right.$$
$$\times \frac{V_0(\mu_n)}{\mu_n} \int_0^{\mathrm{Fo}} q_1(\theta) \exp(\mu_n^2 \theta)\, d\theta + \frac{\mathrm{Bi}_0 r_0}{R} \int_0^{\mathrm{Fo}} \psi_0(\theta) \exp(\mu_n^2 \theta)\, d\theta +$$
$$+ \frac{r_0}{\lambda} \int_0^{\mathrm{Fo}} q_0(\theta) \exp(\mu_n^2 \theta)\, d\theta + \frac{\mathrm{Bi}_1 V_0(\mu_n)}{\mu_n} \int_0^{\mathrm{Fo}} \psi_1(\theta) \exp(\mu_n^2 \theta)\, d\theta +$$
$$\left. + \frac{R^2}{\mu_n \lambda} \int_0^{\mathrm{Fo}} \int_{r_0/R}^{1} rQ(r, \theta) V_0(\mu_n r) \exp(\mu_n^2 \theta)\, dr d\theta\right] \exp(-\mu_n^2 \mathrm{Fo}), \tag{11}$$

where

$$\Phi(\mu_n) = 2\mu_n \left\{\left[\mu_n^2 + \left(\mathrm{Bi}_0 + \frac{R}{r_0}\right)^2\right] \left(1 - \frac{r_0}{R}\right) + \left(\mathrm{Bi}_0 + \frac{R}{r_0}\right)\right\} +$$
$$+ \left[\mu_n^2 - \left(\mathrm{Bi}_0 + \frac{R}{r_0}\right)^2\right] \sin\left[2\mu_n\left(1 - \frac{r_0}{R}\right)\right] - 2\mu_n\left(\mathrm{Bi}_0 + \frac{R}{r_0}\right) \cos\left[2\mu_n\left(1 - \frac{r_0}{R}\right)\right].$$

Similar solutions are obtained for a plate and a hollow cylinder. A whole series of particular solutions can be obtained from these expressions for no fluxes or steady fluxes, solid bodies, etc. for bodies of classical shape.

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SOLUTION OF A NONLINEAR DIFFERENTIAL EQUATION ARISING IN HEAT-TRANSFER THEORY

S. I. Prokopets and L. S. Nyukalova

UDC 532.51

Many problems of the heat-transfer in an incompressible liquid in which an exponential temperature dependence of the viscosity is assumed and mechanical-energy dissipation is taken into account reduce to the same nonlinear (more precisely, quasilinear) differential equation

$$y'' + \frac{1}{2x} y' + e^{y} = 0. \tag{1}$$

This equation is also encountered in astrophysics and chemical kinetics.

In the special case of one-dimensional Poiseuille flow in a plane channel with the channel walls at the same temperature, physical considerations show that the temperature (and velocity) profile is symmetric about the middle of the channel, and the maximum velocity and temperature correspond to the channel axis. Therefore, it is natural to attempt to find a solution of Eq. (1) that is positive, symmetric, and finite on the channel axis. The solution of the Cauchy problem

$$y'(0) = 0, \; y(0) = 1 \tag{2}$$

for Eq. (1) has the desired properties.

The solution of this problem will be sought in the form

$$y(x) = 1 + \sum_{k=1}^{\infty} a_k x^k. \tag{3}$$

Substituting Eq.(3) into Eq.(1) gives, after using Eq. (2), the following recurrence formula for the calculation of the coefficients

$$a_{2k+1} = 0 \quad (k = 0, 1, 2, 3, \ldots), \quad a_0 = y(0) = 1, \; a_2 = \frac{-e}{3},$$

$$a_{2k} = -\left[a_{2k-2} + \frac{1}{2!} \sum_{n=1}^{k-2} a_{2n} a_{2k-2n-2} + \ldots + \frac{1}{(k-1)!} a_2^{k-1} + \frac{1}{(k-2)!} \times \right.$$
$$\left. \times \sum_{n=1}^{2} a_{2n} \sum_{j=1}^{3-n} a_{2j} \ldots \sum_{r=1}^{k-2-n-j} a_{2r} a_{2k-2n-2j-\ldots-2r-2} \right] \frac{e}{k(4k-1)}, \; k = 2, 3, 4, \ldots \tag{4}$$

Hence, the solution of Eq. (1) is the series

$$y(x) = 1 - \frac{e}{3} x^2 + \sum_{k=2}^{\infty} a_{2k} x^{2k}, \tag{5}$$

where the coefficients a_{2k} are calculated from Eq. (4).

Using the method of [1], it is simple to demonstrate the convergence of the series obtained and to calculate the radius of convergence. For x = 1/2 the following majorant series is obtained:

$$1 + \frac{1}{2^2} + \sum_{k=2}^{\infty} \frac{1}{(2k)^2}. \tag{6}$$

Repeating the computations of [1] almost without modification, it may be shown that the radius of convergence is not less than 1/2 in any case and that the series in Eq. (5) converges uniformly, is positive, and is continuous in the region considered, as are its derivatives. While the convergence of the solution is ensured for a finite interval, the solution obtained may be used over a fairly broad range of the thermodynamic parameters of the problem. The uniqueness of the solution is shown analogously.

Note finally that this method may also be used to construct the solution of the boundary problem for Eq. (1).

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CLASS OF APPROXIMATE ANALYTIC SOLUTIONS OF NONLINEAR HEAT-CONDUCTION EQUATION

O. N. Shablovskii

UDC 536.24.02

For the heat-conduction equation in a plane two-dimensional region the first boundary problem of [1] is considered

$$c(T)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda(T)\frac{\partial T}{\partial y}\right],$$
$$x_1\leqslant x\leqslant x_2,\; y_1(x)\leqslant y\leqslant y_2(x),\; 0\leqslant t\leqslant t_0, \tag{1}$$
$$c(T)=\frac{k}{b+T},\; \lambda(T)=l(b+T)^{\varkappa},\; 1+\varkappa\neq 0,$$

$$b+T(x,\,y,\,t)|_{x=x_i}=\left[\frac{\psi_i(z)}{t+\beta}\right]^{\frac{1}{1+\varkappa}},\; z=y+\alpha\ln(t+\beta),\;\beta>0, \tag{2}$$

$$b+T(x,\,y,\,t)|_{y=y_i(x)}=\left[\frac{\Phi(x,\,\tau_i)}{t+\beta}\right]^{\frac{1}{1+\varkappa}},\;\tau_i=y_i(x)+\alpha\ln(t+\beta), \tag{3}$$

$$b+T(x,\,y,\,t)|_{t=0}=\left[\frac{\Phi(x,\,z_0)}{\beta}\right]^{\frac{1}{1+\varkappa}},\; z_0=y+\alpha\ln\beta,\; i=1,\,2. \tag{4}$$

Here T is the temperature; c is the specific heat; λ is the thermal conductivity of the medium; t is the time; x, y are Cartesian coordinates; α, β, $\varkappa$, b, k, and l are constants.

Within the framework of the specified class of solutions the temperature at $x = x_i$ may be defined a priori in the form in Eq.(2); the temperature field at the initial moment and at the boundaries $y = y_i(x)$ is defined arbitrarily by four functions of a single argument.

The investigation of the initial problem reduces approximately to the construction of a solution of the two-dimensional Poisson equation

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial z^2}+\frac{k}{l}=0,\; b+T(x,\,y,\,t)=\left[\frac{\Phi(x,\,z)}{t+\beta}\right]^{\frac{1}{1+\varkappa}},$$
$$\Phi(x,\,z)|_{x=x_i}=\psi_i(z),\;\Phi(x,\,z)|_{z=z_i(x)}=\varepsilon_i(x),\; i=1,\,2,\; 1+\varkappa\neq 0, \tag{5}$$
$$x_1\leqslant x\leqslant x_2,\;[\tau_1,\,\tau_2]\subseteq[z_1,\,z_2].$$

The functions $\varepsilon_i(x)$, $z_i(x)$, i = 1, 2, in this case are arbitrary, and their choice determines the form of the functions $\Phi(x, z_0)$, $\Phi(x, \tau_i)$ in Eqs.(3) and (4); the functions $\psi_i(z)$ appear in Eq. (2) and are given. It is assumed that all the functions are differentiable a sufficient number of times.

The method of [2] is used to solve Eq. (5). The choice of the constant α for the given interval $z\in[\tau_1, \tau_2]$ is such that $|\alpha\partial/\partial z| \ll 1$, the necessary condition for the transition from Eq. (1) to Eq. (5).

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TEMPERATURE FIELDS IN CYLINDRICAL BODIES WITH THERMAL CONTACTS AT THE SURFACE

Yu. I. Malov and L. K. Martinson

UDC 536.24

In calculating the thermal conditions of elements of complex structure, it is necessary to take into account the presence of large numbers of thermal contacts at the surface of each element, through which heat transfer between them occurs. An effective method is proposed for the solution of contact problems of steady heat conduction; in the method, the corresponding boundary problems are reduced to infinite algebraic systems for which approximate methods of solution have been developed.

1. The first step is to obtain the temperature distribution in a cylindrical shell with an annular sector as its cross section. On the side wall of the shell there is an arbitrary number of thermal contacts separated by thermally insulated intervals. Through the thermal contacts, which have different heat-transfer coefficients, heat transfer occurs between the shell and external bodies at various temperatures. The temperature field in the shell is determined by solving the boundary problem for the Laplace equation with boundary conditions of the second kind on the thermally insulated sections of shell surface and boundary conditions of the third kind on the sections of surface bounding the thermal contacts.

2. The temperature distribution in a cylinder of rectangular cross section with symmetrically positioned pairs of thermal contacts on the side wall of the cylinder is now obtained for the case when there is a linear heat source on the cylinder axis. The temperature field in the cylinder is found by solving the mixed boundary problem for the Poisson equation with boundary conditions of the second kind on the thermally insulated sections and boundary conditions of the third kind on the sections bounding the thermal contacts.

3. The temperature distribution in a rod of rectangular cross section is found in the case when a longitudinal heat flux is created by a temperature difference between the ends of the rod, and heat is removed through a discrete system of cooling ribs positioned on two parallel side walls of the rod. The two other side walls are thermally insulated. The determination of the temperature field in the rod reduces to the solution of a mixed boundary problem for the Laplace equation with boundary conditions of the first kind at the end walls and boundary conditions of the second and third kind on the side walls. The results are shown in Fig. 1 in the form of isotherms in the central longitudinal cross section with an asymmetric distribution of thermal contacts (cooling ribs) with the same heat-transfer coefficient. For clarity, the thermal contacts are omitted from Fig. 1.

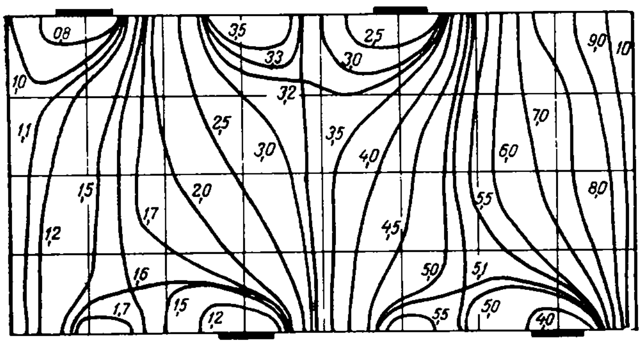

Fig. 1. Thermal field in longitudinal cross section of rod.

4. To solve contact heat-conduction problems, the set of conditions of the second and third kind specified on individual sections of the boundary region is represented in the form of a generalized boundary condition for the temperature u

$$\frac{\partial u}{\partial n} + \theta(s)\, u = T(s), \; s \in \Gamma, \tag{1}$$

where $\theta(s)$ and T(s) are piecewise-constant functions equal to zero on the thermally insulated sections of the boundary Γ and to some constant value depending on the heat-transfer coefficient on the sections bounding the thermal contacts. The solution of the boundary problems of steady heat conduction with the boundary condition in Eq.(1) is written in the form of series expansions with respect to complete orthogonal systems of functions that are particular solutions of the differential equations. The coefficients of these expressions are determined from infinite systems of linear algebraic equations. Estimates show that the matrix operators of the infinite system are Fredholm, and therefore these systems may be solved by the reduction method.

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HEATING OF STEEL CYLINDER IN LIQUID CAST IRON, TAKING INTO ACCOUNT THE TEMPERATURE FIELD IN A DEVELOPING SURFACE LAYER

S. A. Krupennikov and Yu. P. Filimonov

UDC 536.242

The heating of an infinitely long steel cylinder immersed in molten cast iron is considered. A layer of solid cast iron forms on the cylinder surface; as the cylinder is heated the rate of growth of this layer decreases to zero, and the layer then begins to melt. To obtain a numerical solution for the matching of the temperature fields inside the cylinder and in the layer of solid cast iron involves very considerable machine time, and so it is desirable to simplify the model to an extent, so as to permit engineering calculations on small computers.

A possible simplification of the problem is to assume quasisteady temperature variation in the solid layer. In this case the system of equations takes the form

$$0 < x < R,\; \frac{\partial T_1}{\partial x} = a_1 \frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial T_1}{\partial x}\right),$$

$$t = 0,\; T_1 = T_0,\; y = 0,$$

$$x = R,\; \lambda_1 \frac{\partial T_1}{\partial x} = \lambda_2 \frac{T_\mathrm{m} - T_{1R}}{R \ln\left(1 + \frac{y}{R}\right)} = \left(1 + \frac{y}{R}\right)\left[\alpha(T_l - T_\mathrm{m}) + \rho_2 Q\frac{dy}{dt}\right] + q,$$

$$q = \frac{1}{2} c_2\rho_2 \left[(T_\mathrm{m} - T_{1R})\left(1 + \frac{3y}{4R}\right)\frac{dy}{dt} - y\left(1 + \frac{y}{2R}\right)\frac{dT_{1R}}{dt}\right].$$

Comparison of the solution of this system on a Nairi-2 computer with the results obtained by solving the problem rigorously show that the simple model may be used for practical calculations without significant loss of accuracy.

NOTATION

R is the cylinder radius;
y(t) is the thickness of solid layer;
a is the thermal resistivity;
λ is the thermal conductivity;
ρ is the density;
c is the specific heat;

α is the heat-transfer coefficient;
T is the temperature;
T_l is the temperature of molten liquid;
T_m is the melting point of cast iron;
Q is the latent heat of fusion of cast iron.

Indices

1 is the cylinder;
2 is the layer of solid cast iron.

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TRAJECTORY OF ZERO ISOTHERM IN DEGENERATE SINGLE-PHASE STEFAN PROBLEM

A. M. Tsybin

UDC 536.2.01

The degenerate single-phase Stefan problem for a semiplane has a self-consistent solution only in one case: when the temperature at the surface $\varphi(\tau) = T_1 = \mathrm{const} < 0$. If $\varphi(\tau) \neq \mathrm{const}$, the trajectory of the single isotherm $\xi(\tau)$ may be determined using a nonlinear integral equation obtained by Grinberg and Checkmareva. For a boundary condition of the first kind at the surface, the appropriate form of this equation for the present problem is

$$\int_0^\infty \mathrm{ch}\,(\sqrt{p}\,\xi(\tau)) \exp(-ap\tau)\, d\tau = \frac{1}{ap} - \frac{\bar{\varphi}(ap)}{aB}, \tag{1}$$

where

$$\bar{\varphi}(ap) = \int_0^\infty \varphi(\tau) \exp(-ap\tau)\, d\tau, \quad \mathrm{Re}\, p > 0,$$

a is the thermal diffusivity, and B is the ratio of the phase-transition enthalpy to the heat conduction of the solid-zone material.

One of the possible algorithms for the solution of this equation is constructed. Essentially, it is as follows: Assuming that $\xi^2(\tau)$ is an analytic function of the time τ and using the representation

$$\xi^{2n}(\tau) = 2n \sum_{m=0}^{\infty} \frac{a_m^{(n)}}{m+1} \tau^{m+1} \quad (n = 1, 2, \ldots,), \tag{2}$$

and the degeneracy condition $\xi(0) = 0$, the following expression may be obtained

$$a_{m+k}^{(k+1)} = 2 \sum_{i=k-1}^{m+k-1} a_i^{(k)} \frac{a_{m-i+k-1}^{(1)}}{m-i+k} \quad \begin{pmatrix} m = 1, 2, \ldots, \\ k = 1, 2, \ldots, \end{pmatrix}. \tag{3}$$

Then series expansion of $\cosh(\sqrt{p}\,\xi(\tau))$, leads, after using Eq. (2), to the result

$$\sum_{k=0}^{\infty} p^k \sum_{i=k+1}^{\infty} a_{i-k+1}^{(i)} \frac{(i-k-1)!}{a^{i-k}(2i-1)!} + \sum_{k=1}^{\infty} \frac{1}{p^k} \sum_{i=k}^{\infty} a_i^{(i-k+1)} \frac{i!}{a^{i+1}[2(i-k)+1]!} = \frac{-p\bar{\varphi}(ap)}{B}. \tag{4}$$

If $\varphi(\tau)$ increases in absolute value as $T_2\tau^s$, where s = 1, 2, . . . , and $T_2 < 0$, equating terms in the same power of p leads to recurrence relations for $\{a_m^{(1)}\}$ (m = 1, 2, . . .).

This algorithm was tested in the self-consistent case (s = 0). For n = 1, as would be expected, Eq. (2) then contains only one term, which conforms to a known transcendental equation.

The recurrence relations have been realized on a computer.

Numerical results are given for the non-self-consistent problem.

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OPTIMAL PARAMETERS OF AN EXTENDED SPIRAL TURBULIZER

G. I. Tarasov and A. A. Khalatov

UDC 532.551

Heat transfer may be accelerated in a heavily loaded heat exchanger by means of a spiral turbulizer lying within a cylindrical pipe, and research on spiral flows of this type has shown [1] that there should be an optimal solution for the design of such spiral turbulizers, which should provide the maximum rate of heat transfer. Theoretical studies have been made of the flow spiraling in a cylindrical channel having a radial gap with various types of spiral turbulizer; the working parameter may be taken as the ratio of the angular momentum of the flow to the axial momentum multiplied by the radius of the pipe. Optimal angles have been calculated for the spiral, and optimal pitch and number of starts, as well as optimal radii for the central body and other parameters, in each case in order to maximize the angular momentum.

Indirect experimental confirmation has been obtained for the theoretical relationships; the extent of turbulization is proportional to the heat-transfer rate, so one expects and finds turning-point behavior of the heat transfer. Measurements have been made on 18 long spiral devices, and an experimental basis has been defined for calculating the maximum heat-transfer coefficient. There is satisfactory agreement between the theoretical and experimental results for the optimal spiral angle. The best values for the angle are related to the form of the curve on account of the interaction with the flow, particularly if the tube has ribs parallel or perpendicular to the axis of the spiral.

The results can be used in designing and calculating short cylindrical pipes containing such turbulizers with rectangular, trapezoidal, or semicircular grooves (Fig. 1), and also spiral devices placed in cylindrical pipes with only small gaps.

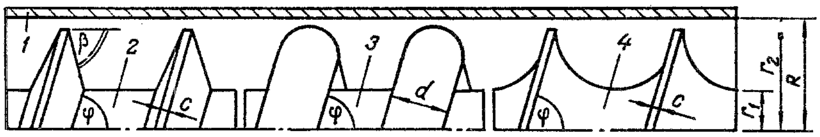

Fig. 1. Types of extended spiral turbulizers: 1) cylindrical channel; 2) spiral with trapezoidal groove; 3) simple wound spiral; 4) spiral with semicircular groove.

NOTATION

R	is the radius of cylindrical pipe;
r_2 and r_1	are the radii of spiral and central body, respectively;
c	is the width of crest of spiral;
d	is the diameter of wound device;
φ	is the winding angle;
β	is the angle of trapezoidal groove.

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AN OPTIMAL NUCLEAR-REACTOR CONTROL ALGORITHM

I. A. Kuznetsov

UDC 621.039.56

The control algorithm for a nuclear reactor involves the exit temperature and hence the reactor power, particularly with a view to minimizing the thermal stress in the components during runup over a given period and over a given temperature range, or else to provide a minimum time to change the power subject to restrictions on the stresses.

The following are the tangential stresses at the surface of a component in a heat-transfer agent of temperature $\theta(\tau)$:

$$\sigma = A\int_0^\tau \frac{d\theta(\tau')}{d\tau'}\,\vartheta(\tau-\tau')\,d\tau'. \tag{1}$$

The kernel of the convolution $\vartheta(\tau)$ is the difference between the mean temperature of the component and the temperature at the surface after unit negative-going temperature discontinuity; $\vartheta(\tau) > 0$ for any τ.

We assume that we have produced a monotonic variation in the heat-transfer agent temperature $\theta_*(\tau)$ such that the maximal thermal stress in the most important component is kept constant at the permissible value σ_0; this then defines the maximum temperature change in a given time or the minimum time for a given change in temperature. In fact, for any $\theta(\tau)$ we have from (1) that

$$\int_0^\tau \left|\frac{d\theta_*(\tau')}{d\tau'}\right|\vartheta(\tau-\tau')\,d\tau' \geqslant \int_0^\tau \left|\frac{d\theta(\tau')}{d\tau'}\right|\vartheta(\tau-\tau')\,d\tau', \tag{2}$$

and for $\vartheta(\tau) > 0$ it follows from any τ that

$$|\theta_*| \geqslant |\theta|. \tag{3}$$

This conclusion can be formulated in a different way: Given temperature changes and given times can be used to produce the minimum thermal stress during the transient response if these stresses are kept constant.

The quasistationary approximation gives the optimum law followed by the coolant temperature as

$$\theta_*(\tau) = -\frac{3\sigma_0}{A\varphi}(1+\varphi\tau). \tag{4}$$

This expression has been written for planar components, and it implies that the minimum thermal stress attainable in the optimal transient response for a given range $\Delta\theta$ or $\Delta\tau$ is

$$\sigma_{\min} = \frac{A\varphi\Delta\theta}{3(1+\varphi\Delta\tau)}. \tag{5}$$

The algorithm for the temperature variation to realize the minimum stress has also been derived for a more accurate expression. Allowance is made not only for the thermal stresses due to the uneven temperature distribution over the cross section but also for those due to deformation of a component as a whole, including the displacement of large volumes during expansion. These stresses arise during startup and shutdown.

NOTATION

τ is the dimensionless time (Fourier number);
Bi is the Biot number;
A is the coefficient of proportionality between the temperature difference and the stresses in the elastic range;
$\varphi = 3\,\mathrm{Bi}/(3+\mathrm{Bi})$.

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Original article submitted December 14, 1976.

HISTORY OF PULSE METHOD OF THERMOPHYSICAL PARAMETER DETERMINATION

A. G. Kharlamov, V. N. Yukovich, and V. I. Krasnov

UDC 536.2.08

The publications on pulse methods have been examined, particularly as applied to the determination of thermophysical parameters such as the thermal diffusivity, specific heat, and thermal conductivity.

The historical development of the method is examined in terms of the numbers of papers appearing in periodicals; this provides an economical way of determining the development scope of thermophysical methods, as well as future prospects for application.

The papers appearing between 1961 and 1974 inclusive have been examined, and the number published each year is constantly increasing which indicates extension of the use of the method. The distribution of the papers by journals indicates that the method is widely used in various branches of applied physics. The numbers of publications by groups indicate the major applications.

The total volume of papers has been compared with exponential and logit curves; at present, the behavior is close to exponential (Fig. 1). Various additional features indicate that this growth tendency should continue for some period. It is also clear that the method has now been largely researched and is becoming widely used in applications.

The temperature range covered by the applications is from 100 to 3300°K; about 70% of the applications relate to the range from room temperature up to 1800°K. The range of measured thermal diffusivities is from $3 \cdot 10^{-3}$ up to 2 cm^2/sec, which shows that the technique is applicable to virtually all solids employed in engineering.

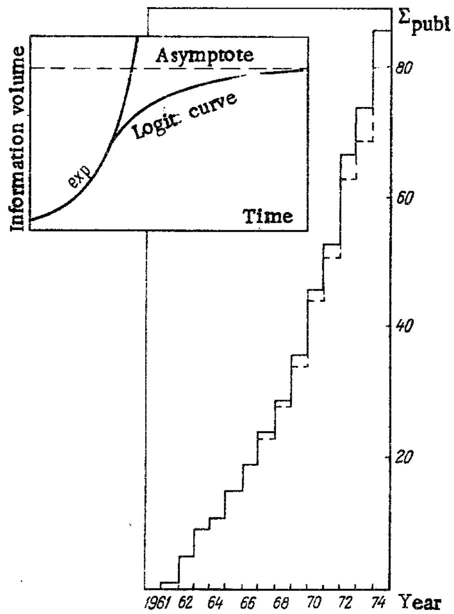

Fig. 1. Diagram of the growth in the number of publications per year. The continuous curve is the growth of the number of publications; the dashed line is the same minus Soviet publications.

Methods are also indicated for systems analysis of the various techniques for determining thermophysical parameters of materials. The approach is readily formalized and allows one to define the place of any particular branch of research in any scientific area where there is an increasing flow of scientific information.

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